

STUDY OF NEUROPSYCHOLOGICAL PROCESSES
AND LOGICO-MATHEMATICAL STRUCTURE
AMONG DYSCALCULICS

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A P P E N D I C E S

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CHAPTER - 1

INTRODUCTION

Concept and General Nature of Dyscalculia.

Dyscalculia is a type of learning disability. Learning disability refers to a retardation, disorder or delayed development in any one or more of the processes of speech, language, reading, spelling, writing or arithmetic. These problems are due to disorder or deficiency in any one or more of the basic psychological processes involved in understanding or in using spoken or written language. They do not include learning problems which are due primarily to visual, hearing or motor handicaps, mental retardation, emotional disturbance or to adverse environmental factors. Thus a child can be considered as learning disabled if (1) he has considerable difficulty in understanding or using spoken language, reading, writing, spelling and/or arithmetic during the developmental period (before 16 years of age), (2) if he is free from visual, hearing or motor disability, mental retardation, severe emotional problems, and (3) he has adequate facilities, interest and motivation to learn basic academic skills.

Herman (1959) proposed that the term acalculia should be used to denote inability to perform calculations; Dyscalculia is a form of acalculia which involves a partial inability to perform calculations.

Some investigators conceive dyscalculia as mathematical disability whereas some others consider it as arithmetic disability. But the behavioural symptoms exhibited by children with mathematical disability are different from that of children with arithmetic disability. Chaffant and Schaffelin (1969) have observed that the child with a mathematical disability might have difficulty - "... handling the operations, interrelations and abstractions of numbers, or the structural, measurement and transformation of space configurations". The child with an arithmetic disability might have trouble "... reading or writing isolated numerals or a series of numerals, reading and writing numbers whose names are not written in the way they are spoken (twenty-one- 21, not 201), recognizing the cat-gorrell structure of numbers (units, tens, hundreds, thousands). and doing computational operations".

In the study, the term dyscalculia is used in a restricted sense indicating arithmetic disability. More specifically it denotes predominant difficulty in

- a) Number concepts - counting and using simple numbers to represent quantity.
- b) Arithmetic processes - adding, subtracting, multiplying, dividing, and,
- c) Arithmetic reasoning - applying basic arithmetic processes in personal and social usage of problem solving (Vallett, 1976).

The child with arithmetic disability, according to Wallace and Kauffman (cited in Fries 1976) specifically exhibits the difficulty in

1. mastering the skills pre-requisite for arithmetic achievement
 - a) discriminating different sizes, shapes and quantities.
 - b) understanding one-to-one correspondance.
 - c) counting
 - d) understanding groups or sets.
2. mastering the basic computational skills and time and money concepts.
 - a) understanding place value.
 - b) fundamental operations - addition, subtraction, multiplication and division.
 - c) understanding fractions
 - d) telling time
 - e) understanding monetary values.
3. problem-solving skills
 - a) understanding arithmetical terms and signs
 - c) the analysis of story problems

Context, Need and Importance.

On the basis of a thorough review of the studies in the area of the education of the handicapped Robert and Anne Marie (1981) pointed out that the past few years have been marked by a growing awareness of the special needs of learning disabled children with respect to mathematics education. They quoted the report of an investigation conducted by Cawley, Fitzmaurice, Goodstein, Lepore, Sedlak, and Athans (1979) about the characteristics of learning-disabled children. The report reveals that different deficit patterns can be noticed among learning disabled children, as a result there is a necessity for the feasibility of different approaches to instruction for the learning disabled child, both from content perspectives and in light of methodological considerations. The significance of

such a recommendation can be realized by understanding the specific characteristics of the population concerned. Considerable amount of work has been done to find out how deviant learning disabled pupils generally are from their normal mates in the relevant neuropsychological processes and cognitive development in terms of Piagetian tasks to determine whether there are qualitative as well as quantitative differences between LD and non-LD pupils in the types of errors committed as well as the kinds of strategies adopted by them while doing arithmetic sums.

In this book 'Teaching Mathematics to the Learning Disabled' Bley and Thornton (1981) (cited in Thomas, 1989) pointed out 11 major behavioural categories in which LD youngsters may deviate from normal development and, as a result, have difficulties in areas of mathematics. They fall into three major categories - Perceptual (Figure-ground, Discrimination, Reversal and Spatial), Memory (Short term and Long-term), Integrative (Closure, Expressive language, Receptive language and Abstract reasoning) processes. Many of the behaviours included, under the "perceptual" category are suggested by theorists who maintain that neurological differences are related to learning disabilities. The behaviours in the memory and integrative categories are the ones cited by many language specialists as contributing factors to learning disabilities. Apart from considering the 11 categories of behaviours the authors have viewed the problem as either a visual or an auditory deficiency. Thomas (1989) noted that these characteristics may not apply to all LD youngsters, and conversely, many of them may be apparent for youngsters who are not learning disabled. This observation reveals three important features of Learning disabled -

- (a) The difficulties noticed among learning disabled children in different academic areas either in isolation or in different combinations (reading and arithmetic; writing and arithmetic; spelling and arithmetic) can be attributed to differential patterns of deficits among them, in the various neuropsychological processes.
- (b) The deficiencies noticed among some youngsters in certain neuropsychological processes may be due to maturational lag. Thus on the basis of such deficiencies, accurate prediction cannot be made with respect to the probability of a child to remain as learning disabled or to become normal.

- (c) The deficiencies noticed among youngsters in certain neuropsychological processes may not lead to academic difficulties, specially the level of deficiency in minimum. This may be explained in two ways - such abilities may not be really required for acquisition of academic skills or the effect of such deficiencies can be considerably reduced through instructional procedures. Whereas the deficiencies in certain other neuropsychological processes really contribute to academic difficulties or their effect cannot be reduced through the traditional instructional practices.

In the light of the above inferences it can be understood that there is a need

- for determining the differences between children with different types of learning disabilities in the relevant cognitive factors.
- for exploring the possible differences among children with any one type of learning disability, say, dyscalculia.
- in conducting follow-up studies in the case of youngsters with deficiencies in certain cognitive factors in order to establish the set criteria which are useful in early detection of children with learning disabilities.
- for establishing a sort of (correlative or causative) relationship between the kinds of difficulties encountered by children with any one type of learning disability or more than that and the deficiencies in specific cognitive factors.
- for devising suitable instructional strategies, predominantly strength-oriented, which are effective in helping the children with learning disability to overcome specific difficulties in any academic area, say, arithmetic.

From the review of the studies in the area of dyscalculia it can be noticed that a sizeable amount of work has been done in comparing learning disabled with non-learning disabled children as well as comparing children with different combinations of learning disabilities with reference to various neuropsychological processes and cognitive development, the kinds of difficulties faced and the types of strategies adopted by them while doing arithmetic sums.

It was consistently showed in the previous studies that mathematically disabled children are poor in ordered recall of items presented in either modality-visual and auditory (Webster, 1979 & 1980), and also in conservation, seriation and classification (Chadwick, 1979 and Derr, 1985). However from the

reports it is not clear whether the samples studied consisted of children with dyscalculia only or other types of learning disabilities also. Hardly there are any attempts to identify subcategories within the group of dyscalculia and to identify differentiating factors among them. There is a real need for initiating and continuing studies in this direction, as such attempts are highly essential in order to understand the real nature of dyscalculia which in turn will form basis for providing remedial instruction to children with such problems. The present study is one such attempt which aimed at identifying whether dyscalculics who are normal in reading and writing are deficient in the cognitive factors mentioned above. It was also further attempted to identify subcategories within the group of dyscalculics and to explore the types of arithmetic difficulties encountered by them and the patterns of deficits they have in the cognitive factors assessed.

CHAPTER - II

REVIEW OF RELATED STUDIES

A review of research in the area of dyscalculia since 1970 upto 1978 reveals that relatively more number of studies were aimed at identifying the neuropsychological characteristics among dyscalculia compared to the other objectives, namely, study of rate of cognitive development, analysing the errors committed and strategies adopted by dyscalculics while doing arithmetic, relating the different types of arithmetic difficulties to underlying cognitive deficits and also remediation of or dyscalculics. However, the number of studies in each of the above mentioned areas are very much limited. Below are given a brief review of such studies. It is difficult to classify the studies under different headings mentioned above as some of them had more than one objectives. So they are broadly classified into following categories for the sake of convenience of discussion. The review of studies related to remediation of dyscalculia is beyond the scope of the study, hence not included.

I. General Nature of Dyscalculia:

Kosc (1974) considers developmental dyscalculia as attributable to hereditary or congenital affection of the brain which affects mathematical functions; this disorder is clearly distinguished from other forms of disturbed mathematical functions. He conducted an investigation of mathematical abilities and disabilities in 375 Czechoslovakian 5th graders. Several tests measuring symbolic functions were administered to 66 suspected dyscalculics with normal intelligence. Results of such a study suggest that nearly 6% of normal children can be justifiably expected to have symptoms of developmental dyscalculia. Kosc considers that developmental dyscalculia is of different forms- verbal, prognostic, lexical, graphical, ideognostical and operational developmental dyscalculia.

Cohn (1971) has observed that development of arithmetical ability is as difficult as that of other elements of language; considerable discrepancy between performances in arithmetic and other language attributes occurs rarely.

Rourke, (1978) proposes that identification of the qualitatively distinct types of disabilities should precede the assessment of the supposed correlates of these problems.

Dunlap, Russell and Ingram (1979) used several reading expectancy formulas to determine arithmetic achievement expectancy for 150 randomly selected 7-12 years old children. Data used were WA based on WISC, CA, IO and Wide Range Achievement Test scores. Correlations between expected scores and achievement were in the .40's for reading but in the 0.70's for arithmetic. Most of the formulae used produced the same results. The investigators concluded that children whose actual achievement is a half year or more below expectancy require diagnostic testing.

II Errors committed and Strategies Adopted by Dyscalculics While doing Arithmetic.

There are many tests and assessment models which can be used to determine a learner's present level of functioning in mathematics. In determining a learner's "present level of functioning" for the purpose of programming, a process often referred to as "diagnostic testing" is followed. (Robert and Anne Marie 1981). In most cases, a battery of tests is given (either criterion-referenced or norm-referenced) and the results are then summarized into a profile or a checklist and suggested teaching objectives are specified. The test is often done individually in a structured situation. Testing procedures are carefully adhered to so that the results can be considered reliable and valid (Robert and Anne Marie 1981)

The common characteristics of different diagnostic models (Robert and Anne Marie 1981) are as follows:

- 1) Emphasis is on error analysis.
- 2) It is more important to understand how a student solves a problem than whether the answer was right or wrong;
- 3) In the construction of problems, teachers need to be aware of problem dimensions and to control for these dimensions when constructing the problems;
- 4) The patterns of errors displayed by learners are not due to carelessness or to insufficient drill but rather are conceptual in nature; and
- 5) Drill in the absence of a corrective instructional sequence is ineffective and quite possibly harmful.

There are several reasons why a student learns to perform an arithmetic skill incorrectly. According to Wallace and McLoughlin (1975) lack of readiness for learning certain arithmetic skills might lead to problems in future. Wallace and

Kauffman (1978) consider that learning problems may be due to failure on the part of the teacher to teach them properly or to give adequate drill of the skill. On the other hand Ashlock (1976) believes that the error patterns that are developed by students are the result of incomplete concept formation. Essentially, the student's overgeneralize a rule that works some of the time, especially with the more elementary forms of problems. As the problems become more complicated or the dimensions of the problems change, the rule is no longer totally effective. The student continues to work problems using the rule because he knows no alternative (cited in Robert and Anne Marie, 1981).

Slade and Russell (1971) discusses about four cases of developmental dyscalculia in adolescents. Subjects were selected on the basis of poor performance on clinical tests involving simple mathematics, complaints of difficulties in mathematics or indications of such from school reports and obtained scores on psychological tests indicating deficits in arithmetical functioning. Results indicate that (a) deficits in arithmetical functions were not accounted for by a low level of intellectual functioning, (b) multiplication was the most deficient process and (c) this deficiency stemmed from poor understanding of multiplication tables, with degree of difficulty varying with each table.

The most common perceptually oriented mathematical errors as noticed by Wagner (1981) are directionality problems, mirror writing, visually misperceived signs (rotational) and directionality (diagonal).

It has been noticed in some studies that children with generalized delayed learning difficulties seldom become proficient in manipulation of numbers. They use the developmental patterns of number expression - large, malformed, bizarre number symbols and stropho symbolic productions- and number operations - utilizing specific graphic elements such as lines, dots, figures and/or fingers, which are frequent among normal children of six-year olds.

Wood (1980) gives five reasons why the learning-disabled child experiences difficulty in learning modern mathematics.

1) The multiple associations of words confuse the child,

2) The lack of mastery of number facts prevents solving problems based on these facts, 3) The symbol notation is also confusing, 4) Teachers are inadequately prepared to teach fundamentals, 5) Parents have been virtually eliminated as adjuncts to the teaching process.

McLeod and Armstrong (1982) surveyed a number of teachers to determine the characteristics of learning disabled youth of secondary age, with respect to mathematics. It was found out that learning disabled youngsters of secondary age generally functioned at the level of upper third to upper fourth grade. The types of problems that were the most difficult for them are:

- Upper level skills of division of whole numbers.
- Basic operations involving fractions.
- Decimals.
- Percents.
- Fraction terminology
- Multiplication of whole numbers.

As these are most basic skill deficits, the investigators concluded that many learning disabled youth are not likely to ever function at grade level in mathematics.

Cawley, Miller, and Schell (1987) explored the arithmetic problem-solving abilities of learning disabled students in Secondary School. They gave problems that differed with respect to two sets of characteristics; direct/indirect or extraneous/non-extraneous. In direct problems, the wording was consistent with the operation to be followed; in indirect problems the wording and the operation were not in agreement. Some problems contained extraneous information and others did not. The data revealed that indirect problems tended to be more difficult for learning disabled students than direct problems and that problems containing extraneous information were more difficult than those without such information. The authors attributed those difficulties to the teaching methodology and the type of problem-solving experiences that are generally emphasized in school curricula.

Englert, Culatta and Horn (1987) also investigated the problem-solving performance of learning disabled students and their peers, in a similar manner. For both the groups of pupils they gave a number of addition word problems with irrelevant linguistic and numerical information embedded within them.

The data revealed that the regular class students exhibited greater accuracy and speed in solving the problems than did their learning disabled peers. Analysis also supported the finding that learning disabled students experience greater difficulties than did non-learning disabled students in problems containing irrelevant numbers. Swanson and Lane (1985) compared the strategy transformations in mathematics of LD and non-LD boys whose mean age was about 13. Performances of the youngsters were compared on seven strategies: reduction to answers, reduction to rule, method replacement, unit building, saving partial results, process elimination and reordering. The data revealed that the two groups differed in their use of strategies to access information and apply it but not in computation. The results also indicated that non-LD children spent more time in considering an alternative method and grouping mental operations and less time recalling computation knowledge than did LD children.

III. Neuropsychological Factors of Dyscalculia:

McLeod and Crump (1978) examined the relationship among selected language and visuospatial measures and mathematics achievement in a group of 43, 6.7 to 11.5 years olds identified as dyscalculics. The following tests were administered to them: content, operations and applications subjects of the Key Math; Diagnostic Arithmetic Test; spatial Relations and Verbal Meaning subjects of SRA Mental Abilities; visual closure, Auditory association and verbal expression sub-tests of ITPA and Right-Left Discrimination sub-test of the Southern California Perceptual Motor Tests. Canonical correlation analysis revealed a general canonical factor permeating both the predictor and criterion variables. The study indicates that verbal ability plays a stronger role in learning disabilities in mathematics.

In a study by Spillacy and Peter (1978) 14 dyscalculic children aged 8-15 years were drawn from a population of learning disabled children and were divided into 2 groups, those with normal or better reading ability and those with dyslexia equal in degree to their dyscalculia. A variety of neuropsychological tests were administered (e.g., Peabody picture vocabulary Test, Bender Gestalt and Embedded figures Test). Both groups showed a variety of behavioural deficits in addition to those comprising Gersfmann's syndrome and were poor in auditory and visual discrimination and motor coordination. Good readers showed severely

impaired ability to make right-left discriminations, while the poor readers were average in this ability. Poor readers showed marked impairment of word fluency and handwriting while good readers were average. The dyscalculia and reading deficits appear unrelated to central-language impairment. Subjects with all 4 elements of the developmental Gerstmann syndrome did not constitute a homogeneous behavioural group and were found among samples of both good and poor readers. The pattern of behavioural deficits shown by these subjects suggests cerebral impairment rather than slow maturation as a probable etiology.

23 Mathematics proficient and 50 mathematics disabled 6th graders were examined by Webster (1979) for short-term memory capacity differences in the recall of serial lists of digits or non-rhyming consonants presented visually or aurally. A Mathematical Group by Input Modality analysis of covariance, with intelligence as the covariate, revealed that the mathematically proficient subjects performed significantly better than the Mathematically Deficient subjects in ordered recall of items presented in either modality, with more information recalled when presented aurally. The Mathematically Deficient subjects recalled more information with the visual presentation. The investigator attributes the deficiency noticed among Mathematically Deficient subjects to the inefficient use of memory encoding strategies during information processing.

In another study Webster (1980) examined the effects of different input-modality/output-modality pairings on the short-form memory of 73, 6th graders identified by their wide Range Achievement Test-results as Mathematics-proficient or Mathematics-disabled. Serial lists of digits and consonants were presented visually and aurally to each subject who then responded either verbally or graphic symbolically during an immediate recall procedure. A significant 2-way Input-Modality by Output-Modality interaction suggested that short-term memory capacity among mildly and severely mathematics-disabled and mathematics-proficient groups differed as a function of the modality used to present the items in combination with the output response required. Like in the case of the above mentioned study the findings are discussed by the investigator relative to the ineffective use of memory codes by the disabled subjects and the usefulness of the modality model as an instructional procedure and curricular rationale.

The study by Burke and Finlayson (1978) (cited in Myklebust, 1978) was an attempt to determine whether children who exhibited arithmetic retardation within the context of differing patterns of reading and spelling performances would also exhibit different patterns of neuropsychological patterns. In this study, learning disabled children between the ages of 9 and 14 years were divided into three groups on the basis of their patterns of performance in reading and spelling tests relative to their level of performance in arithmetic. The subjects in group-1 were uniformly deficient in reading, spelling, and arithmetic. Group-2 was composed of subjects whose arithmetic performance, although clearly below age-expectation was significantly better than their performances in reading and spelling. The subjects in group-3 exhibited normal reading and spelling and markedly impaired arithmetic performance. Although all three groups performed well below age-expectation in arithmetic, the performances of groups 2 and 3 were superior to that of group 1; groups 2 and 3 did not differ from one another in their arithmetic performance. The three groups were equated for age and full scale IQ on the WISC. The three groups performances on 16 dependent measures were compared, and the major findings are as follows: a) the performances of groups 1 and 2 were superior to that of group 3 on measures of visual-perceptual and visual-spatial abilities; and b) group-3 performed at a superior level to that of groups 1 and 2 on measures of verbal and auditory-perceptual abilities.

The study by Brogan and Hardy (1982) verified whether identical and reverse pattern recognition skills would be related to tested reading and arithmetic achievement in 20, 1st graders who attended a remedial reading class. Subjects were given the Ishihara colour Blind Test and the Metropolitan Achievement Test. It was noticed that (1) Same pattern recognition was significantly higher than reverse pattern recognition, (2) Identical pattern recognition did not affect performance on reading or arithmetic achievement, and (3) Reverse pattern recognition significantly affected performance on reading and arithmetic achievement.

IV. Cognitive Development and Dyscalculics.

Pinchman and Meltzer (1976) compared the arithmetic achievement of learning-disabled children with that of normal achievers. 24, 7-8 years old boys served as subjects. The 2 groups, learning-disabled and normal achievers were matched according to age, IQ and Socioeconomic status. All subjects were individually tested on the Key Math, Diagnostic Arithmetic Test and 2 number conservation tasks. Results indicate a group difference in arithmetic achievement but not in the ability to conserve, although a significant correlation was obtained between conservation and arithmetic scores in the learning disabled group. The investigators discuss the findings in terms of the perceptual, memory and attention deficits of learning disabled as well as the possible effects of training on the acquisition of conservation.

Clarke and Chadwick (1979) attempted to test Piaget's theory and its application to learning disabilities and to obtain a diagnostic instrument for dyslexia and for difficulties in arithmetic. 3 groups of children were studied - subjects with problems in reading and writing, subjects with problems in arithmetic, and controls. Piaget's tests were applied, both operational ones (conservation, seriation and class) and figurative ones (representation of Euclidean space, projective space and representation of kinesthetic images). Subjects with arithmetic problems demonstrated a deficient logical-mathematical structure whereas dyslexic subjects demonstrated deficiency in figurative symbolism.

In the opinion of Childs (1961) the elements of arithmetic readiness are classification, correspondence, conservation, reversibility and seriation. In the light of the findings of the above studies this conception appears to be sound.

In a study by Kinine (1984), Duchlander partners and 1st graders (mean ages 52.8 - 82.4 months) completed Piagetian tests (seriation, conservation and multiple-classification), initial arithmetic tests (number-line comprehension and number-language tests). The culture fair Intelligence Test - form 1, and Intelligence subtests from the Primary Mental Abilities (PMA) Test 5-7. It was noticed that from among the Piagetian tests, the combination of conservation and seriation was clearly superior to the intelligence sub-tests from either the culture fair or the PMA in predicting number-language seriation and the combination

Of these intelligence sub-tests were equally good predictors for number line comprehension. From the data the investigators have inferred that out of the Piagetian tasks, seriation might especially serve as a valuable diagnostic instrument for some aspect of initial mathematical education to the traditional intelligence tests.

Nishi (1978) has studied cognitive development of learning disabled children in terms of Piagetian tests. There were 15 arithmetically disabled children of the age group 5.6 to 9.6 years in the study. They were deficient in tasks like object recognition, conservation of length, volume, area and mass, judgment of speed and decoding symbols even at the age of 9.6 years of age.

Derr (1975) investigated the development of the cognitive stages of concrete operations, specifically conservation, in 22 learning disabled children (9-12 years) who had significant deficits in mathematics achievement and in eighteen age matched controls with average achievement in mathematics. Subjects were tested in six areas of conservation namely - 2 dimensional space, number, substance, continuous quantity, weight and discontinuous quantity. Significant group differences appeared, showing that nearly fifty percent of the learning disabled subjects were still non-conservers even in the upper elementary grades.

A three year longitudinal investigation was conducted on the development of conservation skills in learning disabled children by Deborah, James and Mark in 1986. Six measures of conservation (space, number, substance, weight, continuous and discontinuous quantity) from the concept assessment kit were administered to 31 newly identified learning disabled students and 31 normally achieving children, during each of the three years. Results indicated that the learning disabled group demonstrated a developmental delay in attaining the stage of concrete operations compared to the non-learning disabled group. However, when this stage was achieved, the learning disabled group appeared to acquire specific concepts at the same rate as normally achieving children. For the learning disabled children total conservation scores and age were more important predictors of academic achievement than was verbal intelligence during all three years of the study. Whereas, for the non-learning disabled group, this was true only during the first year. It appeared that delayed transition between preoperational and concrete operational thoughts may be an important factor in understanding the continued school failure of learning disabled children.

CHAPTER -III

METHODOLOGY

OBJECTIVES:

- To construct an Arithmetic Diagnostic Test for Primary School Children.
- To identify dyscalculics who are free from dyslexia and dysgraphia from among children studying in primary schools.
- To find out whether there are subcategories within the group of dyscalculics who are normal in reading and writing in terms of the difficulties encountered by them while doing arithmetic sums.
- To analyse the kinds of arithmetic errors committed by such children.
- To find out whether the children with only dyscalculia are deficient in the specific neuropsychological processes - auditory sequential memory (Memory for auditorily presented digits) and Visual sequential memory (Memory for shapes in sequence) and in the different components of logico-mathematical structure - seriation, conservation and classification.
- To find out whether dyscalculics who are normal in reading and writing demonstrate different patterns of deficiencies in the cognitive abilities assessed.

The details regarding methodology - sample, criteria, tools and techniques, methods of analysis of data, interpretation of results are discussed with reference to each objective separately.

A. Construction of an Arithmetic Diagnostic Test for Primary School Children.

There was a need for constructing an Arithmetic Diagnostic Test for School children as a suitable test which is helpful in identifying dyscalculics was not available. The test was developed with the intention of identifying the strengths and weaknesses among children of grades I through IV of Primary Schools while doing different arithmetic tasks appropriate to the respective grades. Thus the test is a criterion-referenced one. The test covers three major areas of Arithmetic namely- Number concept, Arithmetic operations/Processes and Problem-solving. Under each of these areas a series of basic understandings and skills expected to be mastered by children of each of the grades were listed out and suitable test items were constructed to verify such a mastery.

The tables below cover the list of the criterion measures - basic understandings and skills and also the serial number of the corresponding test items which assess them, as included in the final form of the test.

TABLE 3.1: Basic Understandings and Skills relating to number concepts and the serial numbers of the items which assess them.

Sl. No.	Criterion measures.	Sl.No. of the test items.
1	Counting	1(a) & (b)
2	Knowledge of numbers and place value - (a) Reading: (i) Integers (upto 4 digit numbers) (ii) Fractions (Limited to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ and mixed fractions involving these fractions). (b) Writing (Integers)	2(a), (b), & (c) 2(d) 3
3.	Sequential reproduction of numbers (Upto 4 digit numbers)	4(a) to (d)
4.	Concept of 'Lesser than' and 'greater than'.	5(a) & (b)
5.	Serialising the numbers in ascending order.	6(a) to (d)

TABLE 3.2: Basic understanding and skills relating to Addition and the serial numbers of the items which assess them.

Sl.No.	Criterion measures	Sl.No. of the Test items
1	Adding single digit numbers	I (a) to (d)
2	Adding Multidigit numbers according to correct sequence of places: (i) Without carryover (ii) With carryover	I (e), (f) I(f), (h), (i), (j) to (p)
3	Meaning of sign '+'	II (a) to (e)
4	Addition of fractions.	II (c)
5	Adding according to place value.	II(a), (b), (d) and (e)
6.	Serialising given numbers in ascending order.	III
7.	Solving simple problems involving (i) Verbal and numerical relations. (ii) Verbal, numerical and spatial relations. (iii) Spatial and numerical relations.	VI 1(a) & (b); 2(a) & (b) 3(a) & (b) 4(a) & (b)

TABLE 3.3: Basic Understanding and skills relating to subtraction and the serial numbers of the items which assess them.

Sl. No.	Criterion Measures.	Sl.No. of the Test items.
1	Subtracting single digit numbers.	I (a) & (b)
2	Subtracting Multidigit numbers according to correct sequence of places: (i) Without transfer (ii) With transfer	I(c), (d), (g) & (h) I(e), (f), (i) & (j)
3	Meaning of Sign '-'	II (1) to (10)
4	Subtracting according to place value.	II (1) to (6)
5	Subtracting fractions.	II(7) to (10)
6	Seriatizing given sums in descending order.	III
7	Mixed addition and subtraction requiring appropriate categorization.	IV (1) to (5)
8	Checking equality and inequality between sums.	V (1) to (5)
9	Solving simple problems involving: (i) Verbal and Numerical relations. (ii) Verbal, Numerical and Spatial Relations. (iii) Spatial and Numerical Relations. (iv) Numerical relations only.	VI 1(a) & (b), 2(a) & (b), 4(a) & (b) 3(a) & (b) 5(a) & (b) VII (a) to (f)

TABLE 3.4: Basic Understanding and skills related to Multiplication and the serial numbers of the items which assess them.

Sl. No.	Criterion Measures.	Sl.No. of the Test Items.
1	Meaning of sign 'X'	Throughout the test of this section.
2	Multiplying single digit numbers by single digit numbers.	I (a) to (c)
3	Multiplying Multidigit numbers according to correct sequence of places by (i) Single digit numbers. (ii) Multiple digit numbers: (a) with '0' (b) without '0'	I(d), (f), (g), (h), (i) & (l) I (e) I (j), (k) & (m)
4	Seriatizing the given sums in an ascending order on the basis of products.	II
5	Mixed addition, subtraction and multiplication requiring appropriate categorization.	III (1) to (6)
6	Solving simple problems involving - (a) Verbal and numerical relations (b) Verbal numerical & spatial relations.	IV 4(a) and (b) IV 1(a) & (b), 2(a) & (b) 3(a) & (b)
7	Checking equality and inequality between sums.	V

TABLE 3.5: Basic understanding and skills relating to Division and the serial numbers of the items which assess them.

Sl. No.	Criterion Measures.	Sl.No. of the Test Items.
1	Meaning of signs \div and $\frac{\quad}{\quad}$	Throughout the test of this section.
2	Dividing numbers which can be perfectly/clearly divided by the given divisor.	I(a),(b) & (c)
3.	Dividing numbers which cannot be clearly divided by the given divisor.	II (a) (b) (c) & (e)
4.	Dividing where divisor is larger than the relevant components of the dividend.	II (b) to (e)
5.	Mixed addition, subtraction and division requiring appropriate categorization.	III 1(a) & (b) to 6. (a) & (b)
6.	Solving simple problems involving verbal and numerical relations.	IV 1(a) & (b) to 4 (a) & (b)

The above tables clearly reveal that due weightage and adequate representation is given to almost all the criterion measures that are appropriate to all the four grades of the primary schools.

A detailed description of the test with reference to the General Nature, Design, Validity, Standardization and Administration procedures are given in the test book-let itself. The test was constructed both in Kannada and English.

B. Identification of dyscalculics

In order to identify dyscalculics 10 primary schools of Kannada medium which are located in Mysore city were selected. The selection of the schools were made depending upon the feasibility to administer various tests. As dyscalculia is independent of regional differences as well as Socio-economic status, there was no need to consider these variables while selecting the sample for the study. However, in order to control the possible effects of certain pupil variables like interest and motivation to learn mathematics, school variables like methods of instruction, Home related variables like parental involvement in the education of the children both private

management and government schools were included in the study. It is usually assumed that children attending private schools have more advantages than those who are studying in government schools with reference to the above mentioned variables which in turn can be attributed to so many other factors. Thus the intention in selecting both private management and government schools is to verify that whether dyscalculia is independent of all these factors. In other words to prove it is an inherent problem and not merely due to environmental factors.

Since the objective was to find out not only children who have dyscalculia but also to identify those with dyscalculia and without dyslexia (reading disability) and dysgraphia (writing disability), a list of children who were poor in Arithmetic but normal in reading and writing were made on the basis of the teachers' opinion. The table below indicates the number of such children in each of the 10 schools included in the study.

TABLE 3.6: Distribution of the children with difficulty in Arithmetic, in the 10 selected schools.

Sl. No.	Types of Schools.	II Grade			III Grade			IV Grade			Grand Total
		B	G	T	B	G	T	B	G	T	
1	Government	4	3	7	7	5	12	6	9	15	34
2	"	3	2	5	3	8	11	5	5	10	26
3	"	-	10	10	-	16	16	-	20	20	46
4	"	9	-	9	15	-	15	20	-	20	44
5	"	5	2	7	6	8	14	7	7	14	35
6	"	5	3	8	4	7	11	11	5	16	35
7	Private	2	6	8	7	8	15	4	9	13	36
8	"	3	7	10	-	14	14	-	16	16	40
9	"	4	4	8	5	7	12	7	5	12	32
10	"	3	3	6	6	4	10	-	15	15	31
Total:		38	40	78	53	77	130	60	91	151	359

Note: B - Boys ; G - Girls ; T - Total.

Surprisingly it can be noticed from the Table 3.6 that the average number of children who were considered as normal in reading and writing but poor in arithmetic from the teachers of both the private and government schools were exactly same, that is $(139/4) = (220/6)$. Further, it can be noticed that relatively more number of girls were rated to be as poor in arithmetic in all the three grades. In addition, it can also be understood that the number of children who are poor in

arithmetic increases along the grade. Though there were 159 children in IV standard who were considered as poor in arithmetic, only those who were considerably poor were selected on the basis of teachers' opinion. There were 100 such children. Thus the total number was 308.

In order to cross-validate the opinion of the teachers that all these 308 children were normal in reading and writing formal testing was done to assess their level of performance in these skills. The speed and accuracy of reading words was assessed through Kannada oral reading test (Jaya Bai, 1958). It is an individually administered one minute scale. The number of correctly read words per minute gives the raw score. The raw score expected for different grades of the primary school I, through IV are 10, 22, 36 and 43 respectively. Since the testing was done in the beginning of the academic year II standard children were considered to be equal to I standard, III to II and IV to III respectively. So while considering the appropriateness of the level of speed and accuracy of children of different grades, the raw scores obtained by them were compared with the mean score for the previous grade. The children who were average or above average in speed and accuracy of reading words were retained for further screening and the others were eliminated.

To those children who were retained in the previous step (N = 288) Reading comprehension test in Kannada (Ramaa, 1985) was administered. It consists of 8 passages, 2 for each of the 4 grades I through IV of the primary schools. The test was administered individually. The child was expected to read these passages and answer the questions pertaining to them. The questions assess the three levels of comprehension, namely, Literal, Reorganization and Inferential. The child was allowed to read as many number of passages as he could. The testing was ceased under either of the two conditions, specified by the author of the test:

1. When the child committed 18 mistakes in any passage,
Or
2. When the examinee failed to answer at least one question of a particular passage although he committed less than 18 errors while reading the same.

Ore score was given for every correctly answered question. Total score was the sum of scores obtained on all the passages. While judging the adequacy of level of comprehension among these children the same procedures adopted in the case of speed and accuracy was followed. In the sense the raw scores were compared with the mean scores expected for children one-year below the present grade. The children who were inadequate in reading comprehension were excluded and others were retained. The number of children retained in this phase were 275.

To find out whether those 275 children were average in writing the words and the passages when dictated, the tests used for assessing the reading skills were only used. The number of words appropriate to the children of one-year below that of the present grades were dictated to them. It was expected that atleast 75% of the words thus written should be correct. The children who did not meet this criteria were eliminated. Thus only 262 children were left-out. For those children the passages suitable to the children of previous grade were dictated and here also atleast 75% of the words were expected to be correct. Only 251 children satisfied this criteria.

Thus it can be understood that out of the 308 children who were rated as average/above average in reading and writing only 251 were confirmed as either average/above average through formal testing of reading and writing.

The Table 3.7 indicates the number of children who were eliminated/retained at the different phases of the formal testing of reading and writing skills.

TABLE 3.7: Number of children eliminated and retained during formal testing of reading and writing skills.

Sl. No.	Criterion Measures	Tools	Reason for re-jection.	Number of children -	
				Eliminated	Retained
1	Speed and accuracy of reading words.	Kannada Oral Reading Test Jaya Bai (1953)	Speed of reading was not appropriate to grade level expected.	20	288
2	Reading comprehension.	Reading Comprehension Test in Kannada, Ramaa (1935)	Inadequate level of reading comprehension.	13	275
3	Writing the words dictated (Read per minute)	Kannada oral reading test, Jaya Bai (1958)	Failed to write the expected number of words dictated.	13	262
4.	Writing the passage dictated.	Reading comprehension test in Kannada, Ramaa (1935)	Failed to write the passage dictated.	11	251

It was further attempted to find out whether those 251 children who were confirmed to be average/above average in reading and writing were deficient in arithmetic skills. For this purpose the Arithmetic Diagnostic Test for Primary School children (Kannada version) developed by the investigator was administered to them individually. The children were continuously supervised. Their doubts were clarified, examples were given wherever necessary, they were encouraged to do the sums and to do the workings on the book-let itself. Strategies adopted by them were also carefully observed and noted. The answer booklets were evaluated and the responses were analysed qualitatively. Through such an analysis it was noticed that almost all the 251 children were below average in the arithmetic skills, but they varied among themselves, considerably with respect to consistency or inconsistency in response, perseverance and the difficulty exhibited in different levels of tasks. The observations are summarized in the Table 3.8.

TABLE 3.8: General tendency of Inappropriate/Inadequate Responses while doing Arithmetic sums and the frequency of children who showed such responses.

Sl. No.	Inappropriate/Inadequate response.	Possible reasons	Number of children who showed such responses.
1	Difficulty in higher order tasks relating to each area of arithmetic tested.	Unfamiliarity of the tasks.	118
2	Inconsistent response to similar kinds of tasks.	Lack of adequate exercise.	45
3	Difficulty only in the recently introduced arithmetic operations.	Lack of sufficient exposure.	23
4	Failure to perform lower order tasks even though capable of handling higher order ones.	Carelessness.	25
5	Incomplete without obvious reason.	Lack of perseverance.	25
6	Consistent failure even in most basic skills.	Disability to learn Arithmetic.	15

The Table 3.8 clearly indicates only in 15 out of 251 cases the arithmetic difficulty exhibited by the children could be attributed to disability in arithmetic that is dyscalculia. Those 15 children were considered as dyscalculics. Thus the prevalence of dyscalculics who are free from dyslexia and dysgraphia is 6% in the population studied. This figure is exactly same to that noticed by Kose (1974). The table below gives the description of the dyscalculics identified in the study.

TABLE:3.9: Description of the dyscalculics.

Sl. No.	in private schools.	No. in Government schools	No. of boys	No. of girls	No. of students studying in different grades		
					II	III	IV
15	8	7	9	6	3	2	10

From the table it is clear that Dyscalculia is independent of the type of school the children are attending. Though the number of girls who had arithmetic difficulty was more than that of boys, dyscalculia is more prevalent among boys than girls. The number of dyscalculics increases along with the grade level as more difficult tasks are expected at the higher grades.

A detailed analysis of the arithmetic difficulties encountered, the types of errors committed by the dyscalculics of the present study was done. The discussion is given in the chapter IV.

The discussion related to the assessment and analysis of the neuropsychological processes and different components of logico-mathematical structure are given in the chapter V.

CHAPTER IV

Kinds of Arithmetic Difficulties Encountered and Types of Arithmetic Errors committed by Dyscalculics.

I. A. Kinds of Arithmetic Difficulties:

As mentioned in the Chapter III a detailed analysis of the difficulties experienced by all the 15 dyscalculics while doing arithmetic sums was made. The purpose was to find out the typical kinds of arithmetic difficulties and also to find out whether there are any definite patterns in terms of the combinations of the difficulties experienced by them. While doing such an analysis the appropriateness of the tasks to each grade was kept in mind. Since the arithmetic test was administered, in the beginning of the academic year the II standard children were considered as similar to I standard, III to II and IV to III respectively as far as the acquisition of arithmetic skills are concerned. Hence the children were expected to perform the tasks appropriate to the grades one year below the present one. The tables 1.1 to 1.5 in the test booklet served as the source of reference. The basic understandings and skills expected to be mastered by the normal children of respective grades were listed out, and the presence or absence of such a mastery was checked in the case of each dyscalculic with reference to each criterion measure. The data was compiled in the form of tables. The sign 'x' represents absence of mastery, blank indicates presence of mastery and 'NA' means not attempted the particular task. Since the number of dyscalculic in II and III standards were very less in number and also the tasks expected to be mastered by them fall into same categories common tables were prepared in their case. Whereas separate tables were prepared in the case of dyscalculics of grade IV as the categories of tasks appropriate to them are considerably more in number.

TABLE 4.1: Presence or Absence of Mastery in Basic Understandings and Specific Skills Relating to Number Concept among Dyscalculics of Grades II and III.

Sl. No. of the cases.	Grade	Counting	Knowledge of reading integers upto 3 digit Nos.	Writing integers (upto 3 digit Nos.)	Sequential reproduction of Nos. upto 3 digit Nos.	Concept of lesser than and greater than	Serializing Nos. in ascending order
1.	II Std.			x			NA
2.	"			x	x		
3.	"			x			NA
4.	III			x	x		x
5.	"		x		x		

TABLE 4.2: Presence of/Absence of Mastery in Basic Understandings and skills relating to addition among dyscalculics of grades II and III.

Sl. No. of the cases.	Grade	Adding single digit Nos.	Adding digit according to correct sequence of place	Multi- Meaning of sign '+'	Adding according to place value.	Solving simple problems involving verbal and numerical relations.
			With carry-over.	Without carry-over.		
1.	II Std.	x	x	x	NA	NA
2.	"		x		x	NA
3.	"			x	NA	NA
4.	III Std.		x	x	x	
5.	"		x	x		NA

TABLE 4.3: Presence of/Absence of Mastery in Basic Understandings and skills relating to Subtraction among Dyscalculics of Grades II and III.

Sl.No. of the cases.	Grade	Subtract- ing single digit numbers.	Subtracting multi digit Nos. accor- ding to correct sequence of places With transfer	Mean- ing of sign '-'	Subtract- according to place value.	Solving simple problems involving verbal & numerical relations.
1	II Std.	x	x	x	x	x
2	"	x	x	x	NA	NA
3	"		NA	NA	NA	NA
4	III Std.		x	x	x	NA
5	"	x	x	x	x	NA

From the Table 4.1, it is clear that the children of Grade II have difficulty in writing integers and seriating numbers in ascending order where as the common difficulty noticed among Grade III children is sequential reproduction of numbers. Considering both the groups it can be understood that writing integers, sequential reproduction of numbers and seriation were the most difficult tasks to them.

The Table 4.2 reveals that the dyscalculics of Grades II and III except case No.1 demonstrated mastery in adding single digit numbers. Thus this task seems to be simpler to them. This can be understood by their mastery in counting and knowledge of numbers (vide Table 4.1). Majority of them experienced difficulty in all the other kinds of tasks relating to addition, which involved the understanding and application of algorithms. They did not know the meaning of the sign '+'. Word problems were not at all attempted by majority of them. Majority of the dyscalculics of Grades II and III did not have the concept of subtraction at all (vide Table 4.3). They added the numbers instead of subtracting even after proper instruction. The child (Case No.4) who could do subtraction of single digit and multi-digit numbers without the necessity to transfer also failed to do tasks involving understanding and application of set of algorithms.

From the analysis it can be inferred that there is no significant variation among the dyscalculics of Grades II and III with reference to the kinds of difficulties experienced by them in arithmetic. As the number was very less this finding cannot be generalized, till further studies with more number of cases are conducted.

TABLE 4.4: Presence or Absence of Mastery in Basic Understandings and Skills relating to Number concept among Dyscalculics of Grade IV.

Sl.No. of the cases.	Knowledge of Nos. & place value						Sequential reproduction of numbers upto 4 digit numbers	Concept of lesser than & greater than	Seriating the numbers in ascending order.
	Counting	Reading Integers upto 4 digit Nos.	Fractions limited to $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ & mixed	Nos. involving these fractions	Writing integers				
1		x	x		x	x			x
2			x		x	x			x
3		x	x		x	x			x
4		x	x		x	x			x
5		x	x		x	x			x
6		x	x						x
7		x							x
8									x
9					x				x
10			x						x

TABLE 4.5: Presence or Absence of Mastery in Basic Understandings and Skills relating to addition among Dyscalculics of Grade IV.

Sl.No. of cases	Adding single digit numbers		Adding multi digit Nos. according to correct sequence of places		Meaning of sign '+'	Addition of Fractions	Adding according to place value.	Seriating given sums in increasing order.	Solving simple problems involving			
	With carry over	Without carry over	With carry over	Without carry over					Verbal & Numerical relations.	Verbal Numerical & Spatial relations	Spatial Numerical Relations	
1					x	x	x*			x	x	
2					x	x	x				x	
3						x	x**				x	
4	x		x		x	x	x				x	
5	x		x		x	x	x**			x**		
6	x											NA
7					x	x		x		x	x	
8						x				x		NA
9	x						x				x	
10						x						x

Note: * - inconsistent. ** - incomplete.

TABLE 4.7: Presence or absence of Mastery in Basic Understandings and Skills relating to multiplication among Dyscalculics of Grade-IV.

No.	Sl.	Meaning of sign 'x', Multiplying single digit Nos. by single digit No.	Multiplying numbers according to correct sequence of places by single digit Nos.	Multiplying multi-digit numbers according to correct sequence of places by multi digit Nos.	Serating given sums in ascending order.	Mixed addition, subtraction, multiplication, division, comparing, ordering, approximation.	Solving simple problems involving Verbal & Numerical relations.	Verbal, Numerical & Spatial relations.	Checking equality & inequality between sums.
1	x	x	x	x	x	x	x	x	x
2		x	x	x	**	**	**	**	**
3	x	*	x	x	x	x	x	**	**
4	x	x	x	x	x	x	x	x	
5		x	x	x	**	NA	NA	NA	NA
6					x	*	x	x	x
7			x	x	x	x	x	x	x
8				x	*	x	x	x	x
9			x	x	x	NA	NA	NA	NA
10		NA	x	x	x	x	NA	NA	NA

NOTE: x* - inconsistent ; x** - incomplete.

TABLE 4.8: Presence or absence of mastery in Basic Understandings and skills relating to division among dyscalculics of Grade-IV.

Sl. No.	Meaning of sign : $\frac{.}{.}$, $\frac{.}{.}$, $\frac{.}{.}$	Dividing numbers which can be perfectly/clearly divided by the given divisor.	Dividing Nos. which can not be clearly divided by the given divisor.	Dividing when divisor is larger than the relevant components of the dividends.	Solving simple problems involving Verbal and Numerical relations.
1			x	** x	NA
2			x	NA	NA
3		NA	NA	NA	NA
4	x	x	x	x	x
5		x	x	x	NA
6			** x	** x	* x
7	x	x	x	x	x
8	x	x	NA	NA	NA
9			* x	* x	NA
10	x	x	x	x	NA

Note: * - Inconsistent. ; ** - Incomplete.

From the Table 4.4 it can be understood that there are two distinct groups among Dyscalculics of grade IV with respect to the attainment of Basic Skills relating to number concept, the first group (cases 1 to 5) has experienced difficulty in Reading, and Writing the integers, Reading fractions, Sequential reproduction of numbers and seriating the numbers in ascending order. Whereas the dyscalculics of second group (cases 6 - 10) have experienced difficulty mainly in seriating the numbers in ascending order.

With reference to tasks relating to addition the difference between the two groups mentioned above is very less (Table 4.5). They differ mainly in tasks involving meaning of sign '+' and adding according to place value. The dyscalculics of Group I failed to do these kinds of tasks, whereas dyscalculics of Group II performed them correctly.

Among the three types of simple problems relating to addition those involving verbal and numerical relations appear to be simpler to both the groups of dyscalculics. The problems involving verbal, numerical and spatial relations were not successfully attempted by all the cases belonging to both the groups. The problems involving spatial and numerical relations seems to be most complex to dyscalculics of both the groups as none of them worked out those problems correctly. This may be partially due to difficulty in reading fractions (vide Table 4.4) and addition of fractions (vide Table 4.5) which are used in these problems. But, at the same time it can be understood that 3 out of 10 dyscalculics, 1 in Group I and 2 in Group II could not do the problems involving verbal numerical and spatial relations suggesting atleast some dyscalculics have difficulty in understanding spatial relations.

As far as the skills relating to subtraction (vide Table 4.6) are concerned, the dyscalculics of both the groups were very poor. They could do only subtraction of single digit numbers and multidigit numbers without the necessity to transfer. In all the tasks involving use of sign '-' and understanding and application of algorithms they totally failed. However 3 out of 5 in the case of group I could be able to check equality and inequality between sums whereas all the 5 in Group II failed in such tasks.

It is interesting to note that simple problems involving verbal and numerical relations were successfully attempted by dyscalculics of first group whereas other group failed in them. Even in the case of problems involving verbal, numerical and spatial relations the number of children who experienced difficulty were more in the second group than in first group. Taking the frequency in both the groups it can be understood that majority (6/10) of dyscalculics experienced difficulty in solving this kind of problems. The problems involving spatial and numerical relations seems to be most complicated here also like in the case of addition sums. This observation can be explained in similar way as in the case of addition that the difficulty in dealing with fractions as well as spatial relations might have contributed to difficulty in this kind of problems. The dyscalculics of both the group were also failed in solving problems involving only numerical relations. This is obvious because of their difficulty in dealing with tasks requiring the necessity to understand and apply algorithms.

Multiplication seems to be very difficult to dyscalculics (vide Table 4.7). Only 2 dyscalculics of Group I could multiply single digits by single digits whereas other 3 could not do even that. The dyscalculics of Group I could do simple tasks of multiplication. Thus here also some group difference persists. In all the other tasks the performance of both the groups were almost similar.

Dyscalculics experience more difficulty in acquiring the skills of division also. Only 4 children 2 in each group could divide numbers which can be clearly divided.

On the basis of the analysis of the difficulties faced by all the 15 dyscalculics the following inferences can be drawn:

- (a) Dyscalculics differ among themselves in acquiring certain most basic concepts and skills. Some of them can learn them whereas others cannot. But anyhow it is not possible to find out sub-categories in terms of arithmetic difficulties.
- (b) Majority of them experience difficulty in reading and writing integers with more than 2 digits.
- (c) Difficulty in sequential reproduction and seriation of numbers with more than two digits can be considered as indicators of Dyscalculia.

- (d) Dyscalculics do not find difficulty in adding single digit numbers even in the lower grades of primary school. Adding multidigit numbers without necessity for carryover differentiates dyscalculics of II and III grades from that of Grade IV as majority of the latter find them easy. Performing tasks involving carryover seems to be difficult to dyscalculics of even Grade IV.
- (e) Solving simple problems of addition involving verbal and numerical relations appears to be difficult to dyscalculics of grades II and III, whereas dyscalculics of grades IV are able to do them. It appears that minimum skills of addition is sufficient to solve such problems in the case of dyscalculics who are normal in reading. Though the problems included some amount of unnecessary information (vide the test booklet) dyscalculics of grade IV did not find difficulty in solving them. This finding slightly deviates from the observation made by Englert, Culatta and Hern (1987) that learning disabled experience difficulty in word problems with irrelevant linguistic and numerical information. As the report do not clearly indicate whether the training disabled of the study were only dyscalculics or other categories also direct comparison between the findings of the both the studies cannot be done.
- (f) Only minimum level of skills relating to subtraction can be noticed among dyscalculics of grade IV whereas even that is absent in the case of dyscalculics of grades II and III.
- (g) Solving simple problems of subtraction involving verbal and numerical relations seems to be difficult to some dyscalculics of even grade IV indicating that dyscalculics vary in mastering problem solving skills involving different kinds of arithmetic operations.
- (h) Dyscalculics have extreme difficulty in fraction terminology and basic operations (addition/subtraction) involving fractions. They also experience serious difficulty in acquiring even simple multiplication and division skills. This finding supports the conclusion drawn by McLeod and Armstrong (1982) that learning disabled youth of secondary age also have difficulty in the above mentioned skills.
- (i) Dyscalculics in general have difficulty in understanding and applying algorithms relating to different arithmetic processes.
- (j) Considerable number of dyscalculics find difficulty in solving simple problems involving verbal, numerical and spatial relations even though they are capable of solving problems without spatial relations. This indicates that some dyscalculics are deficient in spatial relations.
- (k) Almost all the dyscalculics of the present study failed to solve problems involving spatial and numerical relations and only numerical relations. This may be attributed to deficiency in pre-requisite skills mainly.

B. TYPES OF ERRORS COMMITTED BY DYSCALCULICS:

Apart from analysing the arithmetic difficulties experienced by dyscalculics it was also further attempted to find out the common types of errors committed by them. The table below shows the types of errors, examples and also the number of dyscalculics who committed such kinds of errors.

TABLE 4.9: Types of errors, examples and number of dyscalculics who committed them.

S1. No.	Types of errors.	Examples	No. of dyscalculi who committed such mistakes.
1.	Rotation of numbers	$5 = 3$ $9 = 00$	1
2.	Reversing the digits.	$12 = 21$ $16 = 61$	2
3.	Reading digitwise (lack the concept of place value)	1008 as one, zero, zero, eight.	9
4.	Writing the numbers as we say.	Four hundred and fifty as 40050	9
5.	Lack the knowledge of carryover.	$\begin{array}{r} 24 \\ 37 \\ \hline 511 \end{array}$	9
6.	Subtracting lower number from the higher number irrespective of the place (simplification of the task).	$\begin{array}{r} 72 \\ 45 \\ \hline 33 \end{array}$	10
7.	Multiplication of numbers in a wrong sequence/ writing the product in a wrong place.	$\begin{array}{r} 125 \times 21 \\ 2410 \\ 125 \\ \hline 2535 \end{array}$ $\begin{array}{r} 125 \times 21 \\ 125 \\ 2410 \\ \hline 14910 \end{array}$	7
8.	Multiplying in between dividing (Lack of concept of division)	$\begin{array}{r} 283 \\ 7 \overline{) 473} \\ \underline{28} \\ 283 \end{array}$	8
9.	Bizzare errors.	$15 - 7 = 1$ $\begin{array}{r} 21 \times 7 \\ 47 \end{array}$ $\begin{array}{r} 10110101 \\ 3 \overline{) 95} \\ \underline{30} \\ 65 \\ \underline{30} \end{array}$	8 7 7

The first type of errors ~~was~~ committed by a child of grade IV. This may be due to deficient visual memory. The second type of errors was observed in the children of grade IV. This can be attributed to deficiency in visual sequential memory and also to lack of concept of place value.

The frequency of dyscalculics who committed the remaining types of errors (3-9) are almost same. They imply that majority of the dyscalculics have difficulty in understanding and remembering the rules that govern different arithmetic operations. This suggests the need for enabling them to become consciously aware of those rules and providing enough opportunity to apply them. Mere drill work may not help them in any way. Verbalization of the relevant principles by the teacher while teaching arithmetic may be more effective than simply showing the method of operating. That means teachers have to enable the children not only to understand how a particular operation is done but also to make them understand why it has to be done in such a way only. This kind of exercise would compensate the deficiency they might have in auditory sequential and visual sequential memory.

CHAPTER - V

SEQUENTIAL MEMORY AND LOGICO-MATHEMATICAL THINKING AMONG DYSCALCULICS.

As discussed in the Chapter I and II it was consistently showed in the previous studies that dyscalculics have severe deficiency in recalling the items presented through both the modalities - visual and auditory in a particular sequence (Webster 1979 & 1980). It was also noticed that dyscalculics demonstrate a deficient logical-mathematical structure (conservation, seriation and class) (Clarke and Chadwick, 1979; Kingma, 1984, Derr, 1985, Deborah, James and Mark, 1986). In the present study also it was attempted to assess these variables. But the purpose was not only to find out whether dyscalculics who are normal in reading and writing are deficient in the above processes in comparison with normal children but also to see whether there are any variations among themselves with reference to these variables. And also to explain the arithmetic difficulties experienced and errors committed by them in terms of the deficiencies in the specific abilities as well as to develop an insight into the implication of their relationship for providing remedial instruction to them.

I. Sequential Memory of Dyscalculics:

The memory for shapes in sequence and the memory for auditorily presented digits in sequence were assessed through visual sequential memory and Auditory Sequential Memory subtests of ITPA (Kirk and McCarthy, 1967) respectively. A brief description of these tests are given in the Appendix-I. The tests were administered according to the procedure recommended in the experimenter's Manual. The raw scores were converted into psycholinguistic age for the purpose of analysis and interpretation. The difference between chronological age (CA) and Psycholinguistic age (PA) for visual sequential memory (VSM), CA and Auditory sequential memory (ASM); and Psycholinguistic ages for VSM & ASM were computed. The Table 5.1 represents such computations for each dyscalculic child separately.

TABLE 5.1: Chronological age, Psycholinguistic ages, for sequential memories and difference between them.

Sl. No.	Chronological age (CA)	Psycholinguistic age VSM	D ₁ CA-PA(VSM)	PA ASM	D ₂ CA-PA(ASM)	D ₃ PA(VSM)-PA(ASM)
1	9-2	4-1	5-1	4-5	4-9	0-4
2	9-4	4-10	4-6	4-10	4-6	0
3	9-4	5-10	3-6	4-8	4-8	1-2
4	8-11	4-10	4-1	4-0	4-11	0-10
5	8-11	3-4	0-7	6-6	2-5	1-10
6	9-3	5-7	3-8	4-2	5-1	1-5
7	9-8	4-10	4-10	3-7	6-1	1-3
8	9-0	4-7	4-5	4-10	4-2	0-3
9	8-7	6-2	2-5	4-10	3-9	1-4
10	8-9	8-4	0-5	5-0	3-9	3-4
11	8-6	4-1	4-5	6-6	2-0	2-5
12	7-10	3-10	4-0	4-8	3-2	0-8
13	7-5	4-10	2-7	3-2	4-3	1-8
14	7-4	2-10	4-6	4-8	2-8	1-7
15	6-9	4-10	1-11	4-2	2-7	0-8

Note: VSM - Visual Sequential Memory

ASM - Auditory Sequential Memory

$$D_1 = CA - PA_{VSM}$$

Chronological Age - Psycholinguistic age for VSM.

$$D_2 = CA - PA_{ASM}$$

Chronological Age - Psycholinguistic age for ASM

$$D_3 = PA_{VSM} - PA_{ASM}$$

Psycholinguistic age for VSM - Psycholinguistic age for ASM

(sign neglected)

From the Table 5.1 it can be understood that the psycholinguistic ages for VSM and ASM are much more below that of CA in all the dyscalculics of the study. The difference between CA and PA for VSM ranges from 0-5 to 5-1 years whereas the difference between CA and PA for ASM ranges from 2.0 to 6.1 years indicating greater deficiency in ASM than VSM. In only one case (Case No.2) the PA for VSM is equal to PA for ASM. In five cases (Case No.1, 3, 11, 12 and 14) out of 15, the PA for ASM is greater than PA for VSM. The mean difference is 1-1 year. In the remaining 9 cases the PA for VSM was more than that of ASM by an average of 1.6 years. The finding of the study is in consensus with the findings of the earlier studies that the performance of mathematically deficient subjects were significantly poorer than mathematically proficient children in sequential memory and they recalled more information with the visual presentation (Webster 1979, 1980).

The difficulty noticed among dyscalculics of the present study in sequential reproduction of the numbers and also the type of errors - reversal of digits within the significant deficiency observed among them in sequential memory. But this arises one important question whether reading and writing skills are independent of auditory and visual sequential memories, as the children of the present study were normal in reading and writing. This question can be answered by analysing the basic difference between the learning of alphabet and learning of numbers. Though alphabet of any language for that matter is taught in a particular sequence, there is no rule that the letters have to be learnt in such a sequence only. Adequacy in VSM and establishing association between particular visual feature of the letters and their names. In spite of deficiencies in these abilities the child can learn the names of letters provided he has sufficient visual-verbal association ability. When it comes to word level, word analysis and synthesis abilities play an important role rather than mere VSM and ASM. Thus the deficiency in visual-verbal association, word analysis and synthesis abilities may lead to dyslexia. This assumption is having empirical evidence also (Ramaa, 1985). In languages like English, of course, VSM/ASM are essential to learn the spelling of words. If there is any deficiencies in these abilities spelling can be taught through methods emphasizing the rules governing word formation. But the situation is entirely different in learning numbers.

The learning of numbers is predominantly governed by memorizing Spatial (Visual) and temporal (auditory) orders of numbers. The numbers and digits within numbers should come in a particular order only. Reading and Writing the numbers should be taught in the established sequence only; there is no alternative. Once the child understands place value he can independently read and write multi digit numbers. But in order to understand place value knowledge of numbers (at-least upto 4 digits) is highly essential. Dyscalculics as observed in many studies including the present one write the numbers as we say. For example one hundred and twenty one as 10021. This may be because they remember 100, 20 and 1 easily, but find difficulty in memorizing the sequence 121. If that is the case the method of teaching sequence of numbers should compensate the deficiencies they have in VSM^{ACT} and ASM. Following suggestions may be useful in providing remedial instruction to dyscalculics:

1. Teaching 1 to 10 through drill work.
2. In order to reduce the burden due to deficiency ~~in~~ VSM children should be instructed to write 1 to 9 in a sequence and go on again, 1 behind each number. The child is thus applying his number knowledge upto '9' and perceiving the similarity in the numbers from 11 to 19. Ofcourse here, they have to memorize the auditory counterparts of these numbers in an order only. Learning this particular sequence (11 to 19) is complicated by two features. Firstly there is no similarity between the numbers 11, 12, 13 and 15 in the way we read them, eleven, twelve, thirteen and fifteen respectively. There is no clue to the child to remember them. Secondly in the second set of numbers 14, 16, 17, 18 and 19 the digit only in the unit place is stressed. That is, we read them as fourteen, Sixteen, Seventeen, eighteen. Since the child listens/recalls these digits first he may write them in the first place (left to right). This procedure is similar to that which is adopted while reading/writing the words. There is one to one correspondence between spatial and temporal sequences. This may confuse the children especially with deficiencies in sequential memory when they have to write digits from right to left sequence. In addition to these complications the way we read the numbers 12, 13 & 15 share commonality with 20, 30 & 50 respectively. That means twelve is similar to twenty, thirteen similar to thirty and fifteen similar to fifty. Deliberate attempts should be made to enable the children to learn these numbers by overcoming all the factors which inhibit them. Making the children to become aware of the factors which inhibit them in learning the numbers verbalizing the similarities

and differences between the way we write and read the numbers and the sequence of numbers may be useful in enabling them to function against the inhibiting factors. As the dyscalculics are of average or above average intelligence their ability to discriminate and generalize should be utilized rather than providing a mechanical drill work. It is quite interesting to know that the dyscalculics of the present study who learnt to read the numbers in Kannada also reverse the digits. Infact in Kannada the numbers 11 and 12 are read differently as in the case of English. But however the temporal order is same as that of spatial order. From 13 to 19 there is a kind of similarity. The suffix 'teen' is having a counterpart in Kannada 'Madhi', but it is a prefix. From 21 onwards it is similar to English. This suggest that distinct linguistic features ('names of numbers') themselves are not responsible for the difficulties faced by the children. They may be predominantly attributed to deficiency in VSM/ASM.

3. After helping the children to overlearn these numbers through meaningful and varied experiences the numbers from 20 onwards be taught without much difficulty. Only thing the teacher has to do is to make the children perceive the similarities between the numbers within a particular set, the way we read and write them. For example 21, 22, 23 as - twenty-one, twenty-two, twenty-three; 31, 32, 33 - thirty-one, thirty-two, thirty-three etc.

4. Exercises should be given where the children are expected to write the missing numbers in a particular sequence. This is also a way of firmly establishing the sequence in their mind and also to facilitate the recall of relevant component of the sequence automatically.

5. Learning to recall and reproduce the numbers in a particular sequence should go along with the understanding that the numbers represent increased quantity of entities along with the sequence; the numbers below a particular number represent lesser value and above that represent greater value.

6. Automatic recall of particular set of numbers in a sequential order, knowledge of the relative quantity of entities denoted by these numbers act as pre-requisites for serializing the numbers in an ascending/descending order. The difficulties noticed by the dyscalculics of the study in seriation can be attributed to deficiency in these pre-requisite skills.

7. The tendency of dyscalculics to write the numbers as they read can be corrected through simple strategies. For Example - If the child writes one-hundred and twenty-six as 100206, he can be asked to write them one below the other as follows -

100
20
6

They should understand of course the correct place where they have to write the numbers. After that

they should be asked to replace '0' by the numbers below them. Thus the next steps would be

100	100	100	126.
20	26	26	
6			

After giving sufficient practice, children should be helped to do such exercises, at the mental level itself, and write the numbers in the expected form.

8. Memorizing multiplication table would certainly be quite challenging to dyscalculics because of their severe deficiency in VSM and ASM. There is no wonder if they fail to reproduce the multiples of numbers even at higher grades of schooling. Allowing them to keep open the multiplication table and refer to it whenever necessary including during examination. Probably is the only method feasible in their cases.

9. The difficulties demonstrated by these children in dealing with operations involving multi-digit number suggest that deficiency in VSM and ASM also affect to considerable extent learning and recalling of the set of rules which govern each kind of operations. The implication is that there should be greater opportunity for the dyscalculics to realize and generalize these rules.

II. Metric relations, Conservation and Classification among Dyscalculics:

The logico-mathematic structure among the dyscalculics of the present study was assessed by administering the Metric Relations and conservation sub-tests of Mysore cognitive Development Status Test (MCDST) (Padmini and Navar) (Yet to be published) and the classification test developed by the investigator. The Metric Relations sub-test assesses seriation of length, area, volume, equidistant point location and distance estimation. The last two tasks were also included in addition to seriation tasks in order to find out whether dyscalculics of present study who experienced difficulty in solving simple arithmetic problems involving spatial relations also have deficiency in spatial relations in the concrete situations. The conservation sub-test assesses judgement of Invariance of numbers, area, length, mass and liquid. For details see Appendix II. The raw scores were retained as such and are given in the Tables 5.2 and 5.3.

TABLE 5-2: Raw scores obtained by dyscalculics of different chronological ages on different tasks of Metric Relations sub-test of MCDST.

Sl. No.	C.A.	Length seriation		Area seriation		Volume seriation		Equidistant point location		Distance Estimation	
		MS	OS	MS	OS	MS	OS	MS	OS	MS	OS
1	9-2	6	3	4	4	4	4	5	4	6	0
2	9-4		6		4		4		1		0
3	9-4		2/2		4		4		2		0
4	8-11		2/2		4		4		2		0
5	8-11		1 1/2		4		0		0		0
6	9-3		6		4		4		2		2
7	9-8		3		4		4		1		0
8	9-0		3/2		4		4		2		0
9	8-7		6		4		4		2		2
10	8-9		2/2		2		4		1		0
11	8-6		3		4		4		3		2
12	7-10		5		4		4		5		0
13	7-5		1 1/2		4		4		0		0
14	7-4		3		4		4		0		0
15	6-9		3		0		2		0		0

TABLE 5-3: Raw scores obtained by Dyscalculics of different chronological ages (CA) on different tasks of conservation sub-test of MCDST.

Sl. No.	C.A.	Number		Area		Length		Mass		Liquid	
		MS	OS	MS	OS	MS	OS	MS	OS	MS	OS
1	9-2	2	2	2	1	2	0	2	0	2	0
2	9-4		2		1		0		0		0
3	9-4		2		1		0		0		0
4	8-11		0		1		1		0		0
5	8-11		2		1		1		0		0
6	9-3		2		1		1		0		0
7	9-8		1		1		0		0		0
8	9-0		1		1		2		0		1
9	8-7		2		1		1		2		0
10	8-9		0		1		1		2		0
11	8-6		0		1		1		0		0
12	7-10		1		2		1		0		0
13	7-5		0		1		0		0		0
14	7-4		0		1		0		0		0
15	6-9		0		1		0		0		0

Note: MS - Maximum Score ; OS - Obtained Score.

It is interesting to note in the Table 5.2 that majority of the dyscalculics of the present study experienced difficulty in seriating the objects - dolls and sticks height-wise and length-wise respectively. They failed to seriate length-wise in many cases. This may be because in the case of dolls, while seriating height-wise the base/reference point is fixed. They had to observe the difference only at the top level. But, in the case of sticks they themselves had to find out the base/reference point and compare at the other end. This shows they do not know how to estimate the length and seriate the objects in terms of length even at the 9.0⁺ years.

Majority of the dyscalculics could seriate the objects in terms of area and volume. Only 2 children in each of the cases had difficulty in them. This suggests that dyscalculics even at the age of 7-4 years of age are able to seriate area-wise and volume-wise.

Only two dyscalculics could identify the equidistant point within the given limits (mid-point) and around a given point. These two tasks are loaded with spatial relations, in which dyscalculics were utterly failed.

Distance estimation also involve spatial relation. The raw scores indicate all the children except 2, totally failed in the tasks.

In fact seriation of objects in terms of different dimensions - length, area and volume involves the estimation of the extent of space occupied by an object and judging whether it is relatively more or less compared to the other objects on any one or more dimensions.

The task of equidistant point location and distance estimation in a way are reverse processes wherein the differences in terms of distance (linear) between objects should be narrowed down to as minimum as possible. The dyscalculics of the present study experienced serious difficulty in dealing with both the kinds of situations especially when only one dimension was involved - length or distance.

The study by Rourke and Finlayson (1978) also revealed that the children who were normal in reading and spelling and markedly impaired in arithmetic performance were inferior to children with arithmetic performance better than them in visual-spatial abilities.

From the above findings it can be inferred that the difficulty of the dyscalculics in dealing with concrete situations would definitely influence their performance in tasks dealing with numbers which are relatively more abstract.

Note: For Table 5.3 refer
page 43.

The table 5.3 clearly reveals that only in the case of judgment of invariance of numbers there were a few conservers even at the age of 8-6 to 9.0 years. Many of them were in the transition stage only. The children below the age of 8-6 years in majority of the cases were non-conservers.

With reference to Judgment of Invariance of Area it can be noticed that only one child was a conserver. All the others were in the transition stage irrespective of the age-range. It appears that there is no relationship between chronological age and ability to judge invariance of length among dyscalculics. The number of conservers were very few whereas the number of children who were transitions and ^{non-}conservers are almost same.

Almost all the dyscalculics of the present study were non-conservers with reference to judgment of mass and liquid are concerned.

The findings of the present study fully conforms with that of the earlier studies. It is very clear that dyscalculics attain concrete operational stage considerably at a very late age compared to normal children.

Teaching of Mathematics involves concretization of the situation. When the children fail to operate at the concrete stage itself, they will not be benefitted by the mathematics instruction through mere concretization.

This suggests several alternatives:

1. waiting till they attain concrete operational stage.
2. Developing arithmetic readiness - classification, correspondance, conservation, reversibility and seriation (Childs, 1981) through specialized programmes.
3. Teaching arithmetic through such procedures which would foster cognitive development and develop arithmetic skills simultaneously.

Among the 3 alternatives, third procedure appears to be appropriate as it serves in reducing the gap between normal children and dyscalculics which inturn would prevent serious negative consequences among them.

The ability of the dyscalculics to classify the numbers/set of numbers/geometric shapes in terms of certain characteristic features like - number of digits, odd and even, ascending

and descending order and number of shaded areas, and to explain the basis for classification was tested through the classification test developed by the investigator. The description is given in the Appendix-III.

The responses of the dyscalculics of the different grades on this test are given in the Table 5.4.

TABLE 5-4: Responses of Dyscalculics on Classification Tests.

Sl. No.	Grade	Classification of numbers according to No. of digits.		Classification of sets of signs according to odd and even numbers of them		Classification of the series of numbers in terms of ascending and descending orders.		Classification of geometric shapes based on the extent of shaded areas		
		C	E	C	E	C	E	C	E	
1	IV	1	1	0		1	0	1	1	
2		1	1	1	1	0		0		
3		1	1	1	0	0		1	0	
4		1	1	0		1	0	1	1	
5		1	1	0		0		1	1	
6		1	1	0		1	1	1	1	
7		1	0	1	1	0		0		
8		1	0	1	1	1	1	1	1	
9		1	1	1	1	1	1	0		
10		1	1	0		0		1	1	
1	III	1	1	0						
2	(only upto 2 digits)	1	1	0						
1	II	1	1	0						
2		0		0						
3		1	1	0						

From the Table 5.4 it can be understood that dyscalculics of grade-II have some difficulty in Item-1 and they totally failed in Item-2. The dyscalculics of grade-III were good at Item-1 but failed in Item-2. In the case of dyscalculics of grade-IV, 2 children could not explain the reason for classification. This suggests that some dyscalculics even at the grade IV experience difficulty in verbalizing the criterion adopted by them for classification of numbers depending upon the number of digits.

Only 50% of the dyscalculics of grade-IV could classify the set of signs in terms of odd and even numbers. Only 4 out of 5 could verbalize the reason.

In the case of classification of numbers based on ascending and descending order also only 50% of the dyscalculics were successful out of which only 30% could explain the basis of classification.

Majority of the dyscalculics (7/10) of grade IV were successful in classifying the geometric shapes into four categories depending upon the number of shaded areas, out of these 7, 6 could explain the criterion for classification.

From the above analysis it can be inferred that categorization of numbers and geometric shapes in terms of certain distinct characteristic features are difficult to some dyscalculics even at the grade IV. Even if they classify they could not verbalize the strategy/criteria. This suggests that they require training not only in understanding the properties of numbers- numerals/concrete situations but also training in verbalizing them which will enhance their ability to generalize.

Important observations made in the study:

1. The criterion measures used in the study to identify dyscalculics are valid; they are helpful in identifying children with severe dyscalculia at different grades of primary school.

2. There is no heterogeneity among the dyscalculics thus identified in terms of the arithmetic difficulties experienced and also the types of arithmetic errors committed by them.

3. It is not possible to identify subcategories among dyscalculics in terms of sequential memory, deficiencies and delay in attaining certain capabilities indicative of concrete operational stage, as almost all of them had deficiency/delay to considerable extent.

4. The remedial arithmetic programme **for** dysealculics should be developed keeping in mind the specific deficiencies among them in the relevant aspects of functioning. The attempt should be made to enable them to learn arithmetic skills inspite of the deficiencies in the underlying cognitive processes.

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B I B L I O G R A P H Y

- ① Ashlock, R.B. "Error patterns in computation; a semi-programmed approach", Columbus, Ohio; Charles E. Merritt, 1976.
- ② Bley, N.S. & Thomson, C.A. (1981); "Teaching Mathematics to the Learning Disabled", Austin, TX: PRO-ED.
- ③ Braşman, Ruth and Hardy, Robert.C. "The Relationship between Arithmetic and Reading Achievement and Visual Pattern Recognition in First Grade Children". Alberta Journal of Educational Research, 1982, (Mar.), Vol.28(1) 44-50.
- ④ Byron P Rourke, "Reading, Spelling, Arithmetic, Disabilities". A Neuropsychologic Perspective. In Helmer R Myklebust (Editor) Progress in Learning Disabilities, Vol.IV, 1978.
- ⑤ Cowley J.F., Fitz-Maurice, A.M., Goodstein, H.A., Lepore, A., Sedlak, R.A., and Althaus, V, "Project Math, Levels I - IV". Tulsa, Okla; Educational Progress Corporation, 1976-77.
- ⑥ Cawley, J.F.; Miller, J.H. & School, B.A. (1987) "A brief inquiry of arithmetic word-problem-solving among learning disabled secondary students. Learning Disabilities Focus", 2(2), 87-93.
- ⑦ Chalpan, J.C., and L.A. Scheffelin "Central Processing Dysfunctions in Children" ; A Review of Research, U.S. Government Printing Office, Washington, D.C., 1969.
- ⑧ Childs, Ronald. E. "Developing Arithmetic Readiness of Young EMR Children", Piaget applied, Pointer, 1981 (Fall) Vol.26(1), 19-21.
- ⑨ Clarke, Nancy and Chadwick, Mariana - "Operative and Figurative aspects of thinking in Children with learning problems in Mathematics and Reading Span", Revista Latinoamericana de psicología, 1979, Vol.II (2), 261-272.
- ⑩ Deborah, L.S., James, D. McKinney and Mask, I. Appillbaum "Longitudinal Development of Conservation Skills in Learning Disabled Children". Journal of Learning Disabilities, May 1986, Vol.19(5) 302-307.
- ⑪ Darr, Alice M, "Conservation and Mathematics Achievement in the Learning Disabled Child". Journal of Learning Disabilities, 1985 (Jun-July) Vol.18(6), 333-336.
- ⑫ Dunlap, William P., Russell, Sheldon N and Ingram Cregg. "Academic Therapy", 1979 (Sep.), Vol.15 (1), 81-85.
- ⑬ Englert, C.S; Culatta, B.E.; & Horn D.G. (1987) "Influence of Irrelevant Information in Addition Word Problems on Problem Solving." Learning Disability Quarterly, 10, 29-36.
- ⑭ Fincham, 7 & Meltzer, L. "South African Journal of Psychology", 1976, Vol.6, 80-86.
- ⑮ Herman, K. "Reading Disability", Charles C. Thomas, Springfield, Illinois, 1959.

(16) Kingma, Johannes. "Traditional Intelligence, Piagetian tasks, and initial arithmetic in Kindergarten and Primary School Grade One". Journal of General Psychology, 1984 (Sept.). Vol.145(1), 49-60.

(17) Kose, Ladislav, "Developmental Dyscalculia" of Learning Disabilities", 1974 (Mar.), Vol.7(3), pp.165-177.

(18) McLeod, Terry M and Crump, W. Donald. "The Relationship of Visuo-spatial Skills and Verbal Ability to Learning Disabilities in Mathematics". Journal of Learning Disabilities, 1978 (Apr.) Vol.11(4), 237-241.

(19) McLeod, T.M. & Armstrong, S.W. (1982), "Learning Disabilities in Mathematics- Skill deficits and remedial approaches at the intermediate and secondary level" - Learning Disability Quarterly, 5, 305-311.

(20) Nishi Mary Mathew, 'A Study of Cognitive Development of Learning Disabled Children', unpublished M.Ed. Dissertation, University of Mysore, 1988.

(21) Ramaa, S. "Diagnosis and Remediation of Dyslexia - an Attempt", Unpublished Ph.D. Dissertation, University of Mysore, 1985.

(22) Slade, P.D. and Russell, G.F. "Developmental Dyscalculia", Psychological Medicine, 1971 (Aug), Vol.1(4), 292-298.

(23) Spellacy, Frank and Peter, Barbara, Cortex, 1978 (Jun), Vol.14(2), 197-206.

(24) Swanson, H.L., & Rhine, B. (1985). "Strategy transformations in Learning disabled children's math performance; Clues to the development of expertise". Journal of Learning Disabilities, 18, 596-603.

(25) Thomas C, Lovitt, "Introduction to Learning Disabilities". Allyn and Bacon, 1989.

(26) Valett, R.A. "Programming Learning Disabilities," Fearon Publishers, Belmont, California, 1969.

(27) Wagner, Rudolph, F. "Remediating Common math errors", Academic Therapy, 1981 (Mar.), Vol.16(4), 449-453.

(28) Wallace, G and J.M. Kauffman. "Teaching Children with Learning Problems".

(29) Wallace, C. & McLougatin, J.A. "Learning Disabilities; Concepts and Characteristics", Columbus, Ohio, Charles E, Merrill, 1975.

(30) Webster, Raymon E. "Short-term Memory in Mathematics- Proficient and Mathematics Disabled Students", Journal of Educational Research, 1979 (May/Jun), Vol.72(5), 277-283.

(31) Wood, Mildren H. "Modern Math and the LD Child". Academic Therapy, 1980 (Jan) Vol.15(3), 279-290.

APPENDIX-I

A. VISUAL SEQUENTIAL MEMORY TEST:

It is a sub-test of Illinois Test of Psycholinguistic Abilities developed by Kirk, McCarthy et al (1968) for children of age group 2.4 to 10.3.

This test assesses the child's ability to reproduce sequences of non-meaningful figures from memory. The sequences increase in length from 2-5 figures. Two trials for each sequence are permissible if needed. The child is shown each sequence of figures for five seconds and then is asked to put corresponding chips of figures in the same order. If subject passes the first trial, no second trial is administered; However, subject is always given a second trial if he fails the first trial. The orientation of individual chips is disregarded in scoring.

The test was administered individually to the children of the present study. The administration, recording and scoring were done according to the instructions given in the Examiner's Manual of the test.

Psycholinguistic Age norms are given for different ages from 2.4 through 10.3. The raw scores were converted to Psycholinguistic age for the purpose of analysis.

B. AUDITORY SEQUENTIAL MEMORY TEST:

It is a sub-test of Illinois Tests of Psycholinguistic Abilities developed by Kirk, McCarthy et al (1968) for children of the age group 2.4 to 10.3.

This test assess the child's ability to reproduce from memory sequences of digits increasing in length from 2-8 digits. The digits are presented at the rate of two per second and child is allowed a second trial of each sequence if he fails on the first presentation. He receives more credit for success on the first than the second trial.

Instructions given in the Examiner's Manual of the test was strictly followed in the administration of the test, recording and scoring of the responses.

Psycholinguistic age norms are given for different ages from 2.4 - 10.3. The raw scores were converted to psycholinguistic age for the purpose of analysis.

APPENDIX-II

MYSORE COGNITIVE DEVELOPMENT STATUS TEST* (MCDST) (Padmini and Nayyar, University of Mysore)

This is a test of cognitive development status developed based on a number of cognitive concepts and operations, judged appropriate to age group 5 to 7 years. This assess 6 areas of concrete operational stage, namely Metric Relations, Spatial Relations, Conservation, Temporal Relations, Signs-Symbols and Belongingness. However in this study only two sub-tests - Metric Relations and Conservation were used. Each sub-test consists of many tasks each in-turn has many situations. The description of the two sub-tests are as follows:

Sub-tests	Test-tasks	Situations	Maximum Credit
I. Metric Relations	1. Length seriation	4	6
	2. Area seriation	2	4
	3. Volume seriation	2	4
	4. Equidistant points location	2	5
	5. Distance estimation	3	6
II. Conservation	1. Judgement of invariance of number	2	2
	2. Judgement of invariance of length	2	2
	3. Judgement of invariance of area	2	2
	4. Judgement of invariance of mass	2	2
	5. Judgement of invariance of liquid	2	2

The test was administered to the subjects individually. While administering the test instructions given in the manual were strictly followed. Each situation was presented in the proper manner, demonstrated wherever necessary, minimum cues were given, sufficient time was allowed to the subject to perform the task upto his satisfaction. The subject was encouraged, positively reinforced and negative reinforcement was avoided throughout the period of testing. Scoring was done according to the scoring instructions given in the manual. Raw scores are retained as such. Details about scoring instructions is given below:

SCORING INSTRUCTION

A) Metric Relations

(1) Length Seriation:

- T₁ Score - 2 : for correct solution.
Score - 1 : if dolls arranged in two sub-series or at least any four dolls correctly seriated or correctly seriated after demonstration.
- T₂ Score - 1 : for correct solution.
Score - $\frac{1}{2}$: if sticks are so arranged that one set of ends is seriated, the other end of the sticks is not seriated or correctly seriated after demonstration.
- T₃ Score - 1 : if both the sticks are correctly inserted.
- T₄ Score - 2 : if both the dolls are correctly pointed out.

(2) Area Seriation:

- T₁ Score '2' or '0' in each case.
- T₂ Accepts 1, 2, 3, 4, 5 or 5, 4, 3, 2, 1 as correct solutions.

(3) Volume Seriation:

- T₁ Score 2 or 0 in each case.
- T₂ Accept 1, 2, 3, 4, 5 or 5, 4, 3, 2, 1 as correct solution.

(4) Equidistant Point Location:

- T₁ Score - 2: if all the four dolls are correctly placed anywhere on the dotted circle (approximation or 1/2 cm is allowed).
Score - 1: if only 2 or 3 dolls are correctly placed.
- T₂ Score - 3: if all the four dolls are correctly placed without demonstration anywhere on the dotted straight line.
Score - 2: if all the 4 dolls are correctly placed with demonstration.
Score - 1: if only 2 or 3 dolls are correctly placed with demonstration.

(5) Distance Estimation:

Score 2 or 0 in each situation.

(C) Conservation:

Score - 1: for correct answer. This holds good in all situations.

APPENDIX-III CLASSIFICATION TEST

The test was constructed to assess the ability of Primary School children I through IV to classify

- i. the given numbers written on cards **separately** according to the numbers of digits (1-4),
- ii. the set of signs (+) written on cards into two groups in terms of the number of signs - odd or even,
- iii. the given series of numbers written on cards into two groups depending upon the order - ascending or descending,
- iv. the geometric shapes - square, circle and rhombus into 4 categories on the basis of the extent of shaded area - whole, $1/4$, $1/2$ and $3/4$ parts.

The test items and procedure of administration are as follows:

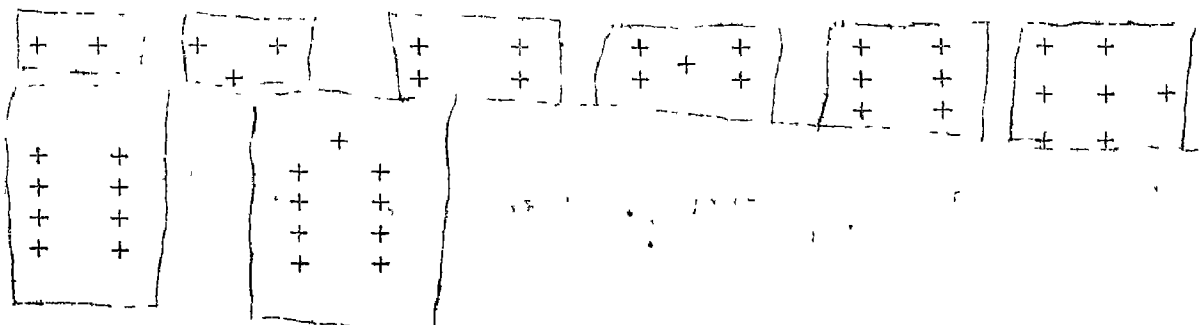
Item 1 (Classification in terms of number of digits).
20 cards with following numbers written on them separately.

1	3	6	7	8
23	44	70	86	95
106	163	474	510	832
2154	2367	3498	4532	5687

The cards were presented to the children in jumbled order. They were asked to classify those numbers into different categories. No clue was given with respect to the criterion of classification. The children were expected to classify them into 4 groups depending upon the number of digits. If the response was correct they were asked to tell the reason why they grouped them like that. The expected reason was 'number of digits'. Ability to classify and to explain the reason were recorded and considered separately.

Item-2: (Classification based on odd and even numbers)

8 cards with set of signs (+) written on them in the following way



It can be noted that the + signs are written in pairs. The extra sign in the case of cards with odd number of signs are written in different places so that the child do not get any clue. He has to identify that some cards contain one extra sign. Here also the cards were presented in jumbled order and the children were asked to make two groups out of them. After correct classification the reason for such a grouping was asked. They were not expected to use the terms 'odd' and 'even'. If they say - in the cards of group I/II, two, two + signs are written together, whereas in the other group there is one single/extra + sign, the reason was considered as valid. Classification in this way, is assumed to be basic to learn the concept of odd and even numbers.

Item-3: (Classification based on sequence of numbers)

There were 6 cards on which the following series of numbers were written:

2	4	5	7	9	4	82	70	67	56	44
8	6	3	3	2	438	442	632	680	375	
27	47	60	62	72	458	399	374	265	262	

The children were asked to observe the cards carefully and make them into only two groups. Here 'two groups' is stressed because there is possibility for them to make three groups depending upon the number of digits like in the case of item-1. The expected response was - in some cards the numbers are written in the increasing order in the remaining in the descending order.

Item-4. (Classifying the shapes into different categories based on the extent of shading).

The item consisted of 12 cards out of which circles squares and rhombus were written on 4 cards each. The size of these shapes were maintained almost same, and $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ parts were shaded in each case. One circle, one square and one rhombus were not at all shaded.

The children were asked to group these cards into different categories. The number of groups were not specified. However it was expected that they should make four

groups depending upon the number of parts shaded irrespective of the shape.

Some children pointed out the number of shaded portion and whereas others pointed out blank portion as the reason for classification. Both of them were accepted as correct answer.

Try out: The test was tried out on 30 children, 10 from each grade of the Primary School II, III and IV. The children were selected on the basis of the teachers' opinion that they were average/above average in Mathematics. The results showed that all the four items were suitable to children of grade IV, items 1, 2, & 3 for grade III and 1 & 2 for grade II. Atleast 80% of them could classify and reason out in the expected manner.

The test was administered to dyscalculics of the present study in the similar way. Only those items which were appropriate to different grades were used. The ability to classify and to verbalize the reason were recorded and considered separately.

